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Variability and probability of annual and extreme precipitation over Serbia and Montenegro

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With 5 Figures

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Summary

Annual precipitation over Serbia and Montenegro is studied in terms of its variability. The dependence of three selected absolute measures of variability (standard deviation, absolute mean deviation and mean absolute interannual variability) from the mean annual precipitation are examined for the area of interest. Two cases of extreme precipitation in Serbia were analysed using the gamma probability density function and some transformations.

1. Introduction

Study of meteorological data collected over a sufficiently long period of time (30 year period is normally required, WMO, 1966) demonstrates various aspects of climate variability. Information about regional precipitation variability and probability is of scientific and practical value, especially in agriculture and hydrology. It is particularly important during recent decades because of detected climate changes (e.g. global temperature rise caused by anthropogenic greenhouse forcing and extreme precipitation, WMO, 2001).

Variability about the mean of a given precipitation series can be expressed in several ways. Generally, it is expressed by the standard deviation and variability coefficient. Studies of variability for precipitation series of different time scales for different regions can be found not only in the current literature (Shin et al., 1990; Lyons, 1990; Tucker, 1993) but also in the older climate literature (Conrad, 1941; Landsberg, 1951).

Conrad (1941) and Nichols (1988) derived a non-linear relationship between the relative variability and mean annual precipitation for a certain number of meteorological stations around the world. For India, the standard deviation and the coefficient of variability for annual rainfall as well as for the summer monsoon are reported by Singh (1984, 1986). Probability analysis of the precipitation series has also been extensively used in different regions of the world resulting in distribution functions (Mielke, 1973; Shenton et al., 1973; Selker and Haith, 1990; Wilks and Eggleston, 1992).

Extremes of meteorological data have an impact on society and so the analysis of climate extremes is important. Long-term daily data can be used for the study of a wide variety of extreme events such as flood producing rains that are of great interest to the general public. The information provided by analysis of extremes includes not only changes in the mean over time but how the statistical distribution of the data changes.

The purpose of the present study is to determine the functional form of the relation between different measures of variability and mean annual precipitation using long-term series of precipitation over Serbia and Montenegro. In our analysis of precipitation series 113 years in length we observed two cases of extreme precipitation in the wider Belgrade area. Namely, during July 1999 and August 2000, annual monthly maximum (AMMAX) and annual monthly minimum (AMMIN) precipitation was observed, respectively. Also, by using the gamma probability density function (PDF) as a standard distribution for estimating extreme precipitation, we evaluated probabilities of the AMMAX and AMMIN (precipitation extremes caused by the possible climatic changes). It is shown that during the 20th century, the temperature and drought index generally increased, extremes increased slightly, while precipitation decreased (WMO, 2001). Further analysis of possible changes in AMMAX and AMMIN precipitation used the Student's t-test.

2. Data analysis

In order to describe the relation of precipitation variability and probability with the mean precipitation over Serbia and Montenegro we used a 45 year period (1951–1995) for annual precipitation data from 35 well distributed meteorological stations (Fig. 1). We also examined the homogeneity and normality of the precipitation series.

2.1 Homogeneity

The homogeneity of the series was tested according to Alexandersson (1986). The standard normal homogeneity test for precipitation is based upon the assumption that the ratio between the value at the station being tested (test station) and a neighboring station (reference station) is fairly constant in time. The central idea was to use the reference stations that are best correlated with the test station. A test parameter T is computed for each of the $N-1$ possible change points in the time series of annual precipitation according to:

$$T(M) = a\bar{P}_1^2 + (N-M)\bar{P}_2^2, \quad a = 1, 2, \ldots, N-1,$$

where N is the number of years, $\bar{P}_1$ is the mean value of P during the M first years, and $\bar{P}_2$ is the mean value during the $(N-M)$ years. The value of M is the year most likely to exhibit inhomogeneity.

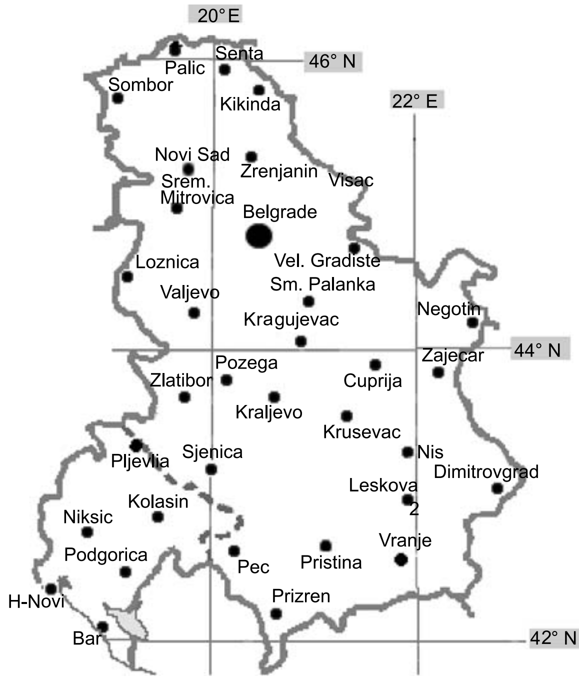


Fig. 1. Map of the territory of Serbia and Montenegro with stations used

The critical T value for the 5% significance levels (T_{95}) is given by Alexandersson (1986). Inhomogeneity was assumed to occur in the year for which T reached its maximum value. In our testing of 35 precipitation series, 33 were found to be homogenous, while Prizren and Herceg-Novi became homogeneous after being adjusted by multiplying their values for the period before the inhomogeneity with an adjustment factor.

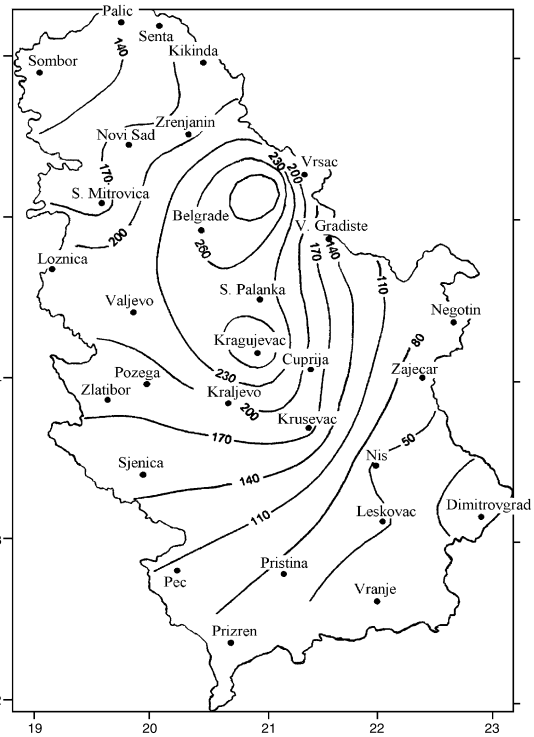
2.2 Normality

Usually in statistical tests of significance – it is required that the data be normally distributed with a constant variance. If the data appear to violate this, a data transformation is often considered. Frequently used transformations that are applied to precipitation data include logarithmic, square-root or the cube-root transformations. In our case, for the 35 meteorological stations in Serbia and Montenegro, during the period (1951–1995) the cube-root transformation was used to achieve normality.

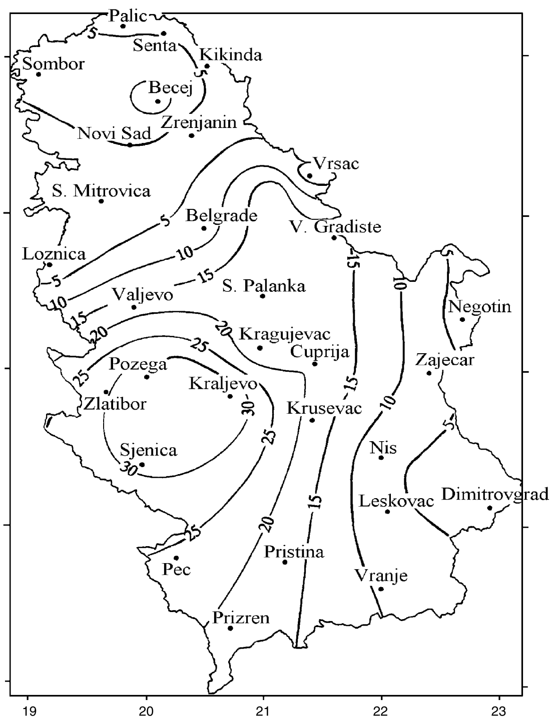
2.3 Precipitation extremes

In this analysis, we separately considered two cases of extreme precipitation in Serbia during

July 1999 and August 2000, which significantly influenced the annual totals. During July 1999 extremely heavy rains (more than 260 mm) that caused floods occurred in the wider Belgrade area (Fig. 2a). August in 2000 was exceptionally dry in the greater Belgrade area and at the north-eastern part of Serbia. The monthly totals were



a



b

Fig. 2. Monthly precipitation sums at Serbia for: **a**) July 1999 and **b**) August 2000

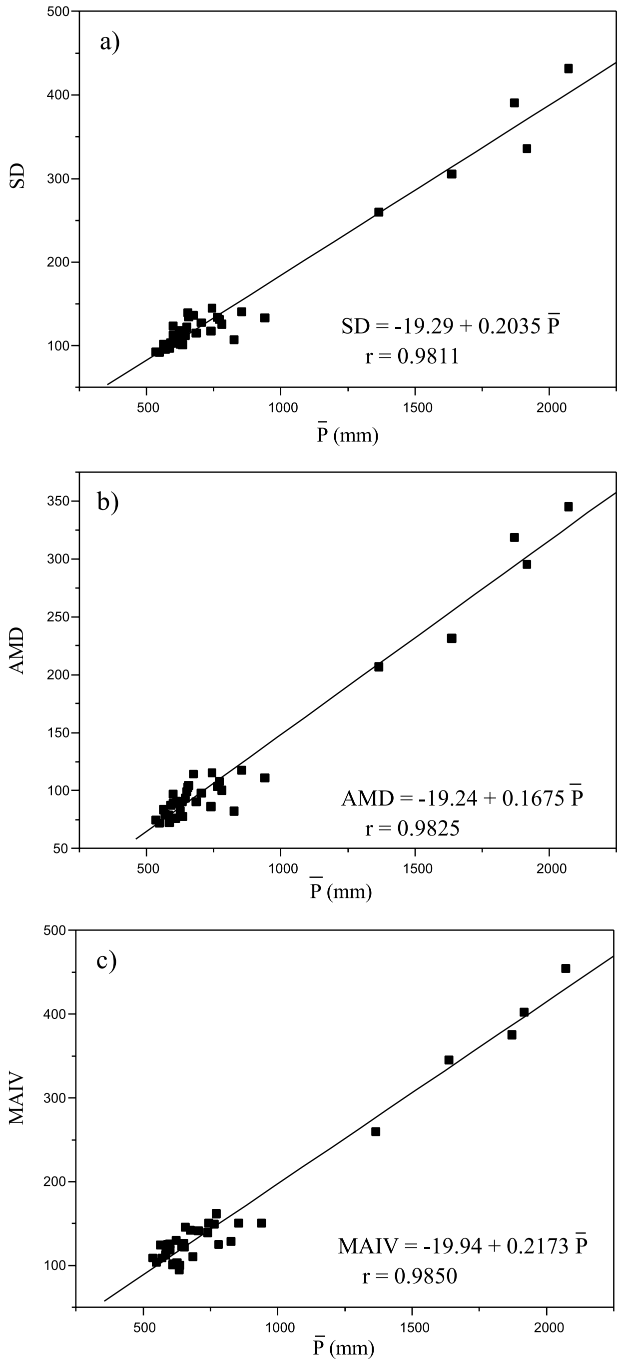


Fig. 3. Scatter diagram of linear regressions between: **a**) standard deviation (SD), **b**) absolute mean deviation (AMD), **c**) mean absolute interannual variability (MAIV) and mean annual precipitation sum ($\bar{P}$); *r* is the correlation coefficient

between 5 and 10 mm, so that August precipitation sums were in the interval of the lowest recorded in these areas (Fig. 2b). It has been mentioned that the area of Belgrade is located in a continental climatic region: the highest monthly precipitation occurs in June, while the minimum occurs in February.

3. Variability and probability analysis

In climatological analysis, precipitation variability and probability are expressed in absolute terms. For analysis we used the standard deviation (SD), absolute mean deviation (AMD) and mean absolute interannual variability (MAIV) as absolute measures of variability.

This analysis shows the relationships between SD, AMD and MAIV with mean annual precipitation ($\bar{P}$). The relationships were found to be linear in the form of:

$$\mathrm{SD} = -19.29 + 0.2035\bar{P},$$

$$\mathrm{AMD} = -19.24 + 0.1675\bar{P},$$

$$\mathrm{MAIV} = -19.94 + 0.2173\bar{P},$$

where the mean annual precipitation is given in mm (Fig. 3).

Also, the correlation coefficient (r) is given in the respective diagram in Fig. 3, along with the linear regression. The high values of the correlation coefficient (0.9811, 0.9825, 0.9850, respectively, Fig. 3) show that the linear relationships between absolute measures of variability and the mean annual precipitation over Serbia and Montenegro are highly significant (above 0.01 per cent level).

For normally distributed global precipitation series Landsberg (1951) found that the ratio of the MAIV and SD was 1.129. The ratio outlined above is 1.079 for all stations in Serbia and Montenegro (Table 1). Hence, our result is compatible with the results of Landsberg.

Table 1. Ratio of mean absolute interannual variability (MAIV) and standard deviation (SD) for the stations at Serbia and Montenegro

N	Station	Height(m)	MAIV/SD
1	Kikinda	81	1.13
2	Palić	102	1.17
3	Srem.Mitrovica	81	1.10
4	Sombor	88	1.19
5	Vel. Gradište	82	1.04
6	Zrenjanin	80	1.14
7	Senta	80	1.22
8	Vršac	84	1.03
9	Ćuprija	123	1.09
10	Kruševac	166	0.88
11	Sm. Palanaka	122	0.88
12	Valjevo	176	0.99
13	Zaječar	144	0.96
14	Loznica	121	1.20
15	Negotin	42	1.08
16	Priština	573	1.12
17	Zlatibor	1028	1.13
18	Pljevlja	784	1.13
19	Novi Sad	84	1.12
20	Beograd	132	0.96
21	Kragujevac	185	1.01
22	Herceg-Novi	10	0.96
23	Bar	4	0.99
24	Podgorica	49	1.13
25	Dimitrovgrad	450	0.99
26	Kraljevo	215	1.11
27	Leskovac	230	0.98
28	Niš	201	1.14
29	Požega	310	1.19
30	Prizren	402	1.04
31	Sjenica	1038	1.11
32	Vranje	432	0.94
33	Peć	498	1.07
34	Nikšić	647	1.07
35	Kolašin	944	1.05
	Mean		1.07

4. Evaluation of extreme precipitation frequency distribution

In many instances, a quantitative evaluation of the ability of the selected probability density function (PDF) to describe adequately the observed extremes (maximum as well as minimum) precipitation frequency distribution has never been addressed before (Sevruk and Geiger, 1981, 1987; Legates, 1991). The statistical analysis of the AMMAX and AMMIN precipitation series for the Belgrade station during the period 1888–2000 (113 years) was performed using a gamma distribution.

The gamma distribution with two parameters is the special case of the Pearson Type III distribution, when the location parameter is zero. Its PDF is given by the equation:

$$f(x) = \frac{1}{\beta^{\alpha}\Gamma(\alpha)} x^{\alpha-1} \exp\left(-\frac{x}{\beta}\right), \quad x>0, \quad \alpha>0, \quad \beta>0.$$

where α and β are the shape and scale parameter, respectively. Γ is the gamma function.

4.1 Two cases of extreme precipitation in Serbia

Indicators based on daily precipitation data during the second half of the 20th century in Europe showed mixed patterns of change, but significant increases occurred in the extreme amounts derived from wet spells and the number of heavy rainfall events (in central Europe). However, most of the precipitation indicators show a significant upward trend in their annual anomalies (WMO, 2001). This observed change of precipitation extremes is in keeping with expected changes due to the enhanced greenhouse effect. For a doubling of CO_2, using the GISS model, Brázdil (1992) showed that in autumn the most dramatic increase in precipitation rates should be in the regions of the Appenine and the Balkan Peninsulas. In our analysis we considered separately AMMAX and AMMIN precipitation data for Belgrade, where the above mentioned extremes were recorded over 113 years.

Each precipitation station represents a given surrounding area. Its representativeness depends on the nature and time scale of precipitation that is considered. Two cases of extreme precipitation in Serbia were caused by wider cyclonic and anticyclonic activities that persisted for several days and almost a month, respectively. Belgrade station may be representative of the wider area of interest, since extreme precipitation was recorded elsewhere during such weather situations.

Testing the fit of a PDF of sampled data to the gamma PDF was done by using the chi-squared goodness of fit test. The parameters of the gamma PDF (α, β) are estimated from the measured date by the moment method. Empirical and gamma PDFs of AMMAX as well as AMMIN precipitation are presented in Fig. 4. The resulting empirical (the gamma) and theoretical χ^2 statistics of AMMAX as well as AMMIN precipitation are shown in Table 2.

According to Table 2, the condition $\chi^2 < \chi^2_{0.05}$ is satisfied by the gamma PDF for both data sets. It is thus clear that AMMAX and AMMIN precipitation at the Belgrade can be taken to be gamma distributed. Öztürk (1981) noticed that the use of the gamma distribution leads to an efficient estimate of the probability of small amounts of precipitation.

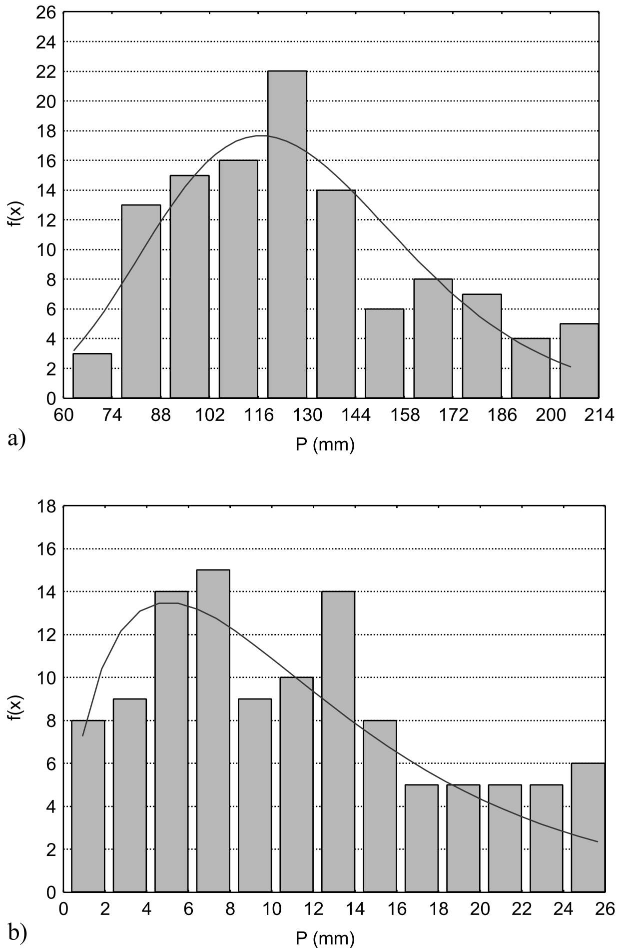


Fig. 4. Empirical and the gamma PDF of: **a**) annual monthly maximum precipitation and **b**) annual monthly minimum precipitation for the Belgrade station during 1888–2000

Table 2. Empirical and theoretical statistics of AMMAX as well as AMMIN precipitation for Belgrade during 1888–2000

Data used	Empirical χ^2 statistics	Theoretical $\chi^2_{0.05}$ statistics
AMMAX prec.	10.332	15.507
AMMIN prec.	13.673	18.307

The normal distribution is an asymptotic case of the gamma distribution as the shape parameter tends toward infinity. According Dingens and Steyart (1971) data are transformed (X) to the normal distribution (Y) by using:

$$Y = 2\left(\frac{x}{\beta}\right)^{1/2}, \quad \text{for AMMAX precipitation and}$$

$$Y = \left(\frac{x}{\alpha\beta}\right)^{1/3}, \quad \text{for AMMIN precipitation,}$$

where α and β are the parameters of the gamma distributions. The results obtained are presented in Fig. 5. The resulting empirical and theoretical χ^2 statistics for the transformed form of the AMMAX and AMMIN precipitation series are shown in Table 3.

Table 3 shows that the transformed form of the AMMAX and AMMIN precipitation series at Belgrade can be taken to be normally distributed.

Further, we used the Student's t-test to determine possible differences between the means in the periods: 1888–1975 and 1976–2000; 1888–1970 and 1971–2000 in Belgrade. This test statistic is determined as:

$$t = \frac{\left(\bar{Y}_2 - \bar{Y}_1\right)}{\left\{\left[\frac{N_1 S_1{}^2 + N_2 S_2{}^2}{N_1 + N_2 - 2}\right]\left[\frac{1}{N_1} + \frac{1}{N_2}\right]\right\}^{1/2}},$$

where $(\bar{Y}_2 - \bar{Y}_1)$ represents the difference in group means of precipitation extreme, N_1 and N_2 are the number of cases within each subsample, and S_1 and S_2 are the standard deviations in the subsamples. When t is outside the bounds of the two-tailed probability of the Gaussian distribution t_g (equal to 1.96 at the 0.95 confidence level), a significant shift in the mean is assumed. The results obtained are represented in Table 4.

The Student's t-test indicates that there are no significant changes in the AMMAX and AMMIN precipitation during the two periods used for examination. In the case of heavy precipitation in the Belgrade area (the highest amount of precipitation of 262.5 mm recorded during 113 years) we obtained that probability of precipitation greater than 260 mm occurring is 0.0014

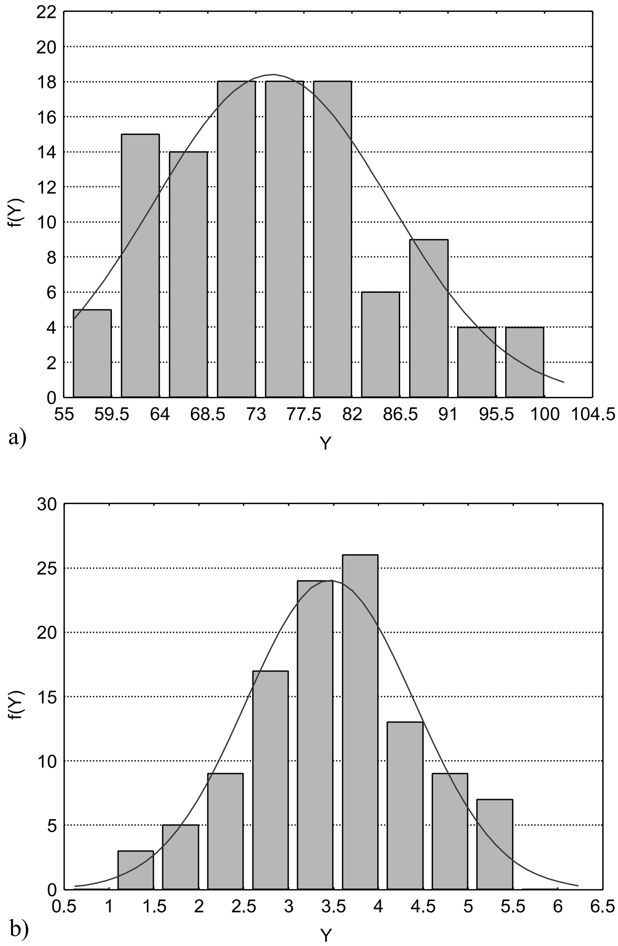


Fig. 5. Transformed form of the gamma to the normal distribution of: **a**) annual monthly maximum precipitation and **b**) annual monthly minimum precipitation for the Belgrade station during 1888–2000

Table 3. Empirical and theoretical statistics for the transformed form of AMMAX as well as AMMIN precipitation for Belgrade during 1888–2000

Transformed form of:	Empirical χ^2 statistics	Theoretical $\chi^2_{0.05}$ statistics
AMMAX prec.	11.016	14.067
AMMIN prec.	6.663	12.592

Table 4. The results of the Student's t-test of the AMMAX and AMMIN precipitation for Belgrade during the two periods examined

Results of the t-test	AMMAX precipitation	AMMIN precipitation	Period 1	Period 2
	0.0612	0.3714	1888–1975	1976–2000 (25)
	0.3135	0.1306	1888–1970	1971–2000 (30)

with the estimated return period exceeding 700 years. In the second case, very low precipitation in the same area (7.8 mm), the probability of precipitation lower than 8 mm is 0.3974, with a return period exceeding 2.5 years.

5. Conclusions

In our analysis of variability of annual precipitation over Serbia and Montenegro and extreme precipitation in the greater wider Belgrade area we can conclude that:

- The magnitude of three absolute measure of variability (SD, AMD, MAIV) increase linearly with the mean annual precipitation;
- The ratio of MAIV and SD calculated for all meteorological stations is 1.079;
- The gamma PDF gives a good fit to the AMMAX and AMMIN precipitation in the Belgrade area;
- The gamma PDF can be transformed to the normal PDF with appropriate relationships;
- The Student's *t*-test indicates that there are no significant changes in AMMAX and AMMIN precipitation during the two periods used for study.

It is becoming increasingly important to understand the nature of extreme precipitation, in order to be able to utilize optimally low and high rainfall areas for agricultural purposes. An analysis of precipitation amounts is of great value in this context, and can provide estimates of probabilities of having more (or less) precipitation than certain specified amounts.

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